# Filter Overview

- 24 State EKF
  - Quaternions (Q0,…,Q3)
  - Velocity (NED)
  - Position (NED)
  - Gyro delta angle bias vector (XYZ)
  - Accelerometer bias (XYZ)
  - Earth magnetic field vector (NED)
  - Magnetometer bias errors (XYZ)
  - Wind Velocity (NE)
- Uses the following sensors
  - Inertial Measurement Unit angular rates and specific forces
  - GPS position (in local NED frame)
  - GPS velocity (in local NED frame)
  - Pressure altitude
  - 3-axis magnetometer

# EKF State Equations

- Pose information is captured in the first 10 states which use a dynamic process model that defines the movement of the body frame (XYZ RH axis system) in a navigation inertial reference frame (North, East, Down)

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \\ V_N \\ V_E \\ V_D \\ P_N \\ P_E \\ P_D \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} \text{Quaternion rotation from body to nav frame} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{North, East, Down velocity} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{North, East, Down position} \end{matrix}$$

# EKF State Equations

- The first four states are the quaternions that define the angular position of the XYZ body frame relative to NED navigation frame.

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

- The rotation matrix from body to navigation frame is given by:

$$[T]_B^N = \begin{bmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2(q_1 \cdot q_2-q_0 \cdot q_3) & 2(q_1 \cdot q_3+q_0 \cdot q_2) \\ 2\,(q_1 \cdot q_2+q_0 \cdot q_3) & q_0^2-q_1^2+q_2^2-q_3^2 & 2(q_2 \cdot q_3-q_0 \cdot q_1) \\ 2(q_1 \cdot q_3-q_0 \cdot q_2) & 2(q_2 \cdot q_3+q_0 \cdot q_1) & q_0^2-q_1^2-q_2^2+q_3^2 \end{bmatrix}$$

- Where the rotation from navigation to **body** frame is required, the transpose will be used and is denoted by $[T]_N^B$

# EKF State Equations

- The truth delta angles are calculated from the IMU measurements and delta angle bias states $\Delta_{ang\_bias}$

$$\Delta_{ang\_meas} = \begin{bmatrix} \Delta_{ang\_x} \\ \Delta_{ang\_y} \\ \Delta_{ang\_z} \end{bmatrix} = \int_{t_k}^{t_{k+1}} \omega \cdot dt$$

$$\Delta_{ang\_bias} = \begin{bmatrix} \Delta_{ang\_bias\_x} \\ \Delta_{ang\_bias\_y} \\ \Delta_{ang\_bias\_z} \end{bmatrix} \text{Delta angle bias states}$$

$$\Delta_{ang\_truth} = \Delta_{ang\_meas} - \Delta_{ang\_bias}$$

# EKF State Equations

- The truth delta velocities are calculated from the IMU measurements and delta velocity states $\Delta_{vel\_bias}$

$$\Delta_{vel\_meas} = \begin{bmatrix} \Delta_{vel\_x} \\ \Delta_{vel\_y} \\ \Delta_{vel\_z} \end{bmatrix} \text{ Delta angle IMU measurements}$$

$$\Delta_{vel\_bias} = \begin{bmatrix} \Delta_{vel\_bias\_x} \\ \Delta_{vel\_bias\_y} \\ \Delta_{vel\_bias\_z} \end{bmatrix} \text{ Delta angle bias states}$$

$$\Delta_{vel\_truth} = \Delta_{vel\_meas} - \Delta_{vel\_bias}$$

# EKF State Equations

- The quaternion $\Delta_{quat}$ that defines the rotation from the quaternion at frame k to k+1 is calculated from the truth delta angle $\Delta_{ang\_truth}$ using a small angle approximation. The inertial navigation uses the exact method.

$$\Delta_{quat} = \begin{bmatrix} \Delta q_0 \\ \Delta q_1 \\ \Delta q_2 \\ \Delta q_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{\Delta_{ang\_truth\_x}}{2} \\ \frac{\Delta_{ang\_truth\_y}}{2} \\ \frac{\Delta_{ang\_truth\_z}}{2} \end{bmatrix}$$

- The quaternion product rule is used to rotate the quaternion state forward by the delta quaternion $\Delta_{quat}$ from frame k to k+1.

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}_{k+1} = \begin{bmatrix} q_0\Delta q_0 - q_1\Delta q_1 - q_2\Delta q_2 - q_3\Delta q_3 \\ q_0\Delta q_1 + \Delta q_0 q_1 + q_2\Delta q_3 - \Delta q_2 q_3 \\ q_0\Delta q_2 + \Delta q_0 q_2 - q_1\Delta q_3 + \Delta q_1 q_3 \\ q_0\Delta q_3 + \Delta q_0 q_3 + q_1\Delta q_2 - \Delta q_1 q_2 \end{bmatrix}$$

# EKF State Equations

- The truth delta velocity vector is rotated from body frame to earth frame and gravity is subtracted to calculate the change in velocity states from frame k to k+1

$$\begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}_{k+1} = \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}_k + [T]_B^N \cdot \Delta_{vel\_truth} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \cdot \Delta t$$

- The position states are updated using Euler integration (the inertial navigation uses a more accurate trapezoidal integration method)

$$\begin{bmatrix} P_N \\ P_E \\ P_D \end{bmatrix}_{k+1} = \begin{bmatrix} P_N \\ P_E \\ P_D \end{bmatrix}_k + \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}_k \cdot \Delta t$$

# EKF State Equations

- The IMU sensor bias, magnetic field and wind states all use a static process model

$$\begin{bmatrix} \Delta_{ang\_bias\_x} \\ \Delta_{ang\_bias\_y} \\ \Delta_{ang\_bias\_z} \\ \Delta_{vel\_bias\_x} \\ \Delta_{vel\_bias\_y} \\ \Delta_{vel\_bias\_z} \\ M_N \\ M_E \\ M_D \\ M_X \\ M_Y \\ M_Z \\ Vwind_N \\ Vwind_E \end{bmatrix}_{k+1} = \begin{bmatrix} \Delta_{ang\_bias\_x} \\ \Delta_{ang\_bias\_y} \\ \Delta_{ang\_bias\_z} \\ \Delta_{vel\_bias\_x} \\ \Delta_{vel\_bias\_y} \\ \Delta_{vel\_bias\_z} \\ M_N \\ M_E \\ M_D \\ M_X \\ M_Y \\ M_Z \\ Vwind_N \\ Vwind_E \end{bmatrix}_k \begin{array}{l} \left.\vphantom{\begin{matrix} a \\ a \\ a \end{matrix}}\right\} \text{IMU delta angle bias} \\ \left.\vphantom{\begin{matrix} a \\ a \\ a \end{matrix}}\right\} \text{IMU delta velocity bias} \\ \left.\vphantom{\begin{matrix} a \\ a \\ a \end{matrix}}\right\} \text{Mag field – navigation frame} \\ \left.\vphantom{\begin{matrix} a \\ a \\ a \end{matrix}}\right\} \text{Mag field – body frame bias} \\ \left.\vphantom{\begin{matrix} a \\ a \end{matrix}}\right\} \text{Wind velocity – nav frame} \end{array}$$

# EKF Observation Equations

- The GPS position, Baro height and GPS velocity involve direct observation of states, so the observation model is trivial.
- The magnetomer is assumed to be aligned with the body frame and experiences a magnetic field vector which is the sum of a navigation frame filed rotted into body frame and a body frame fixed field.

$$\begin{bmatrix} M_X \\ M_Y \\ M_Z \end{bmatrix}_{meas} = [T]_N^B \cdot \begin{bmatrix} M_N \\ M_E \\ M_D \end{bmatrix} + \begin{bmatrix} M_X \\ M_Y \\ M_Z \end{bmatrix}_{bias}$$

- We can also use the earth frame magnetic field declination as an observation

$$\psi_{\ DECLINATION} = \tan^{-1}\left(\frac{M_E}{M_N}\right)$$

This can be used to prevent unwanted yaw rotation of the earth field estimates during periods when heading is poorly observable.

# EKF Observation Equations

- We can also use the rotation matrix elements to provide a direct yaw observation model using either a 321 or 312 Euler sequence

$$\psi_{321} = \tan^{-1}\left(\frac{2(q_1 \cdot q_2 + q_0 \cdot q_3)}{q_0^2 + q_1^2 - q_2^2 - q_3^2}\right)$$

$$\psi_{312} = \tan^{-1}\left(\frac{-2(q_1 \cdot q_2 - q_0 \cdot q_3)}{q_0^2 - q_1^2 + q_2^2 - q_3^2}\right)$$

By selecting the appropriate transformation, a direct heading measurement can be used that avoids gimbal lock.

# EKF Observation Equations

- The optical flow observation equation assumes a sensor aligned with the Z body frame at a distance R from a stationary scene in the navigation frame.

$$\begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix} = [T]_N^B \cdot \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}$$

$$\begin{bmatrix} LOS_X \\ LOS_Y \end{bmatrix}_{meas} = \begin{bmatrix} \dfrac{-V_Y}{R} \\ \dfrac{V_X}{R} \end{bmatrix}$$

- The visual odometry  observation equation assumes measurement of velocity states rotated into the body frame:

$$\begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix}_{meas} = [T]_N^B \cdot \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}$$

# EKF Observation Equations

- The airspeed observation equation assumes a sensor that measures the magnitude of velocity relative to the wind field:

$$\begin{bmatrix} Vrel_N \\ Vrel_E \\ Vrel_D \end{bmatrix} = \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} - \begin{bmatrix} Vwind_N \\ Vwind_E \\ 0 \end{bmatrix}$$

$$TAS_{meas} = \sqrt{\left({Vrel_N}^2 + {Vrel_E}^2 + {Vrel_D}^2\right)}$$